These are three fundamental Laws of geometrical optics. (9)
(eq^n 34, 35, 36)

Yet, we have used only generic form (eq^n 29), not applied any specific boundary conditions.

Any other waves → water waves, sound waves etc.
↳ expected to obey the same "optical" Laws when they pass from one medium to other.

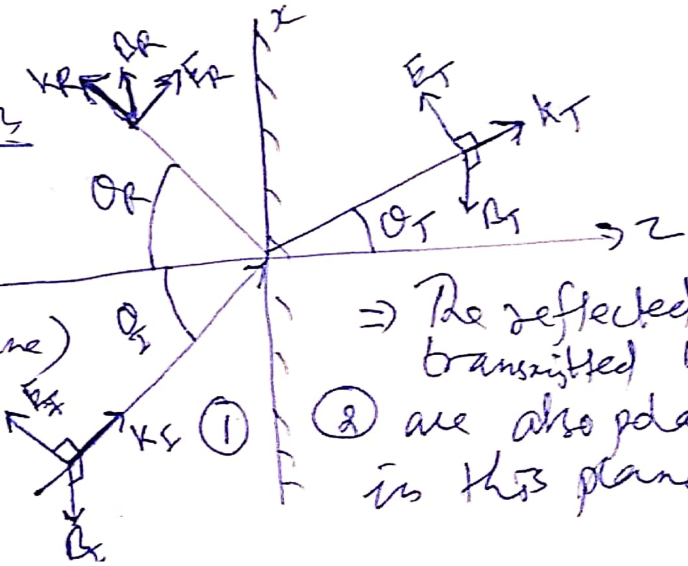
Given eq^n (30), the boundary conditions eq^n (10) become:

$$\left.\begin{array}{ll} \text{(i)} & \epsilon_1(\tilde{E}_{0I} + \tilde{E}_{0R})_z = \epsilon_2(\tilde{E}_{0T})_z \\ \text{(ii)} & (\tilde{B}_{0I} + \tilde{B}_{0R})_z = (\tilde{B}_{0T})_z \\ \text{(iii)} & (\tilde{E}_{0I} + \tilde{E}_{0R})_{x,y} = (\tilde{E}_{0T})_{x,y} \\ \text{(iv)} & \frac{1}{\mu_1}(\tilde{B}_{0I} + \tilde{B}_{0R})_{x,y} = \frac{1}{\mu_2}(\tilde{B}_{0T})_{x,y} \end{array}\right\} \quad \text{--- (37)}$$

where $\tilde{B}_0 = \left(\frac{1}{v}\right)\hat{k}\times\tilde{E}_0$ in each case.

→ Pairs of equations, one for the x-component and one for the y-component.

Suppose that the polarization of the incident wave is parallel to the plane of incidence (the xz plane)



⇒ The reflected and transmitted waves are also polarized in this plane.

eq^n (i) reads

$$\epsilon_1 \left(-\tilde{E}_{0I}\sin\theta_I + \tilde{E}_{0R}\sin\theta_R\right) = \epsilon_2\left(-\tilde{E}_{0T}\sin\theta_T\right) \quad \text{--- (38)}$$

(ii) $\rightarrow$ no contribution, magnetic fields have no z components ($0=0$)

(iii) becomes $\tilde{E}_{0I}\cos\theta_I + \tilde{E}_{0R}\cos\theta_R = \tilde{E}_{0T}\cos\theta_T$ --- (39)

and (iv) $\rightarrow$

$$\frac{1}{\mu_1 v_1}\left(\tilde{E}_{0I} - \tilde{E}_{0R}\right) = \frac{1}{\mu_2 v_2}\tilde{E}_{0T} \quad \text{--- (40)}$$

Given the laws of reflection and refraction, eq^n (38), (40) reduce to

$$\tilde{E}_{0I} - \tilde{E}_{0R} = \beta\,\tilde{E}_{0T} \quad \text{--- (41)}$$

where

$$\beta \equiv \frac{\mu_1 v_1}{\mu_2 v_2} = \frac{\mu_1 n_2}{\mu_2 n_1} \quad \text{--- (42)}$$

and eq^n (39) reduces to

$$\tilde{E}_{0I} + \tilde{E}_{0R} = \alpha\,\tilde{E}_{0T} \quad \text{--- (43)}$$

where $\alpha \equiv \dfrac{\cos\theta_T}{\cos\theta_I}$ --- (44)

Solving eq^n (41) & (43) for reflected and transmitted amplitudes, we obtain

$$\boxed{\tilde{E}_{0R} = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)\tilde{E}_{0I}, \quad \tilde{E}_{0T} = \left(\frac{2}{\alpha+\beta}\right)\tilde{E}_{0I}} \quad \text{--- (45)}$$

Fresnel's eq^n $\rightarrow$ for the case of polarization in the plane of incidence.

Transmitted wave ⟶ always in phase with incident one

Reflected wave ⟶ either in phase if $\alpha > \beta$

or 180° out of phase if $\alpha < \beta$

α ⟶ function of θ_I

↳ amplitude of the transmitted and reflected waves depend on the angle of incidence:

$$\alpha = \frac{\sqrt{1-\sin^2\theta_T}}{\cos\theta_I} = \frac{\sqrt{1-\left[\underbrace{\left(\frac{n_1}{n_2}\right)\sin\theta_I}_{\text{Snell's Law}}\right]^2}}{\cos\theta_I}$$

In the case of normal incidence $(\theta_I = 0)$, $\alpha = 1$ (46)

$$\Rightarrow \boxed{\tilde{E}_{0R} = \left(\frac{1-\beta}{1+\beta}\right)\tilde{E}_{0I}, \quad \tilde{E}_{0T} = \left(\frac{2}{1+\beta}\right)\tilde{E}_{0I}}$$

At grazing incidence $(\theta_I = 90^\circ)$, α diverges, and the wave is totally reflected

At an intermediate angle, θ_B (called Brewster's angle) ⟶ reflected wave is completely extinguished

According to eq^n (45), this occurs when $\alpha = \beta$, or or $$\sin^2\theta_B = \frac{1-\beta^2}{(n_1/n_2)^2-\beta^2} \quad \text{---(47)}$$

From eq^n (46)

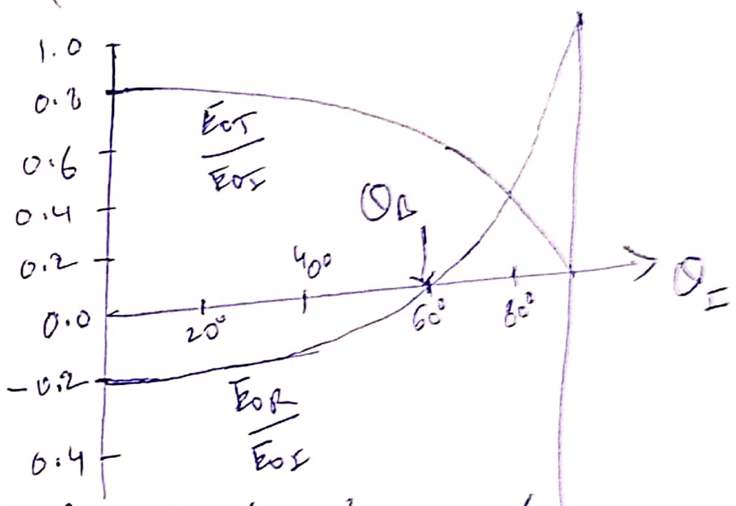
$$\left[\beta^2 = \frac{1-(n_1/n_2)^2\sin^2\theta_B}{1-\sin^2\theta_B}\right]$$

For the typical case $\mu_1 \cong \mu_2$, so $\beta \cong \frac{n_2}{n_1}$

$$\sin^2\theta_B \cong \frac{\beta^2}{1+\beta^2}$$

$$\Rightarrow \boxed{\tan\theta_B \cong \frac{n_2}{n_1}} \quad \text{--(48)}$$

Transmitted and reflected amplitudes as functions of θ_I for light incident on glass ($n_2 = 1.5$) from air ($n_1 = 1.0$) $\longrightarrow$



negative number $\longrightarrow$ wave is 180° out of phase with the incident beam

Power per unit area striking the interface $\rightarrow \mathbf{S}\cdot\hat{z}$

Incident intensity

$$I_I = \frac{1}{2}\epsilon_1 v_1 E_{0I}^2 \cos\theta_I \quad \text{--(49)}$$

Reflected and transmitted

$$I_R = \frac{1}{2}\epsilon_1 v_1 E_{0R}^2 \cos\theta_R \quad \text{and} \quad I_T = \frac{1}{2}\epsilon_2 v_2 E_{0T}^2 \cos\theta_T$$

--(50)

Cosine terms → We are considering average power per unit area of the interface → Interface is at an angle with wave front.

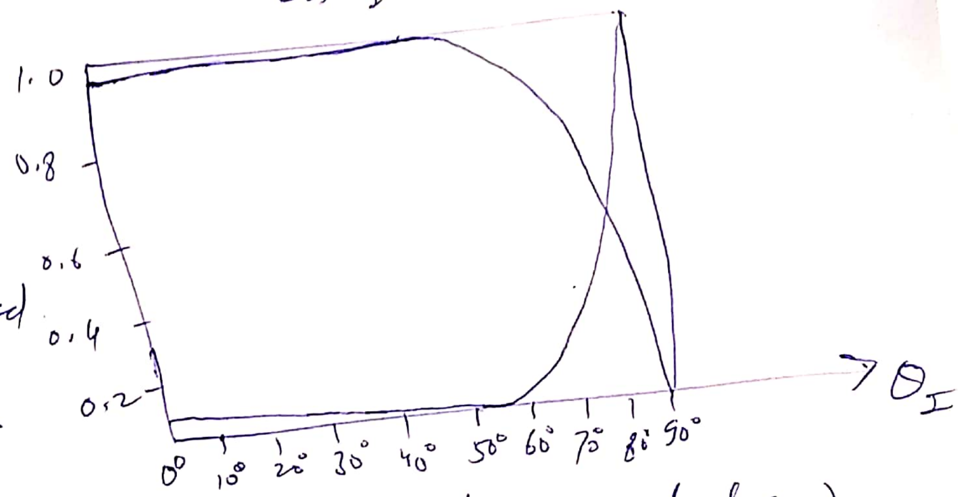
Reflection and transmission coefficients for waves polarized parallel to the plane of incidence are;

$$R = \frac{I_R}{I_I} = \left(\frac{E_{0R}}{E_{0I}}\right)^2 = \left(\frac{\alpha-\beta}{\alpha+\beta}\right)^2 \quad \text{--- (51)}$$

$$T = \frac{I_T}{I_I} = \frac{\varepsilon_2 v_2}{\varepsilon_1 v_1}\left(\frac{E_{0T}}{E_{0I}}\right)^2 \frac{\cos\theta_T}{\cos\theta_I} = \alpha\beta\left(\frac{2}{\alpha+\beta}\right)^2 \quad \text{--- (52)}$$

R → goes to zero at Brewster's angle.

T → fraction transmitted
↳ goes to 1 at θ_B



(for air/glass interface)

$R + T = 1$

Conservation of energy
↳ The energy per unit time reaching at a particular patch of area on the surface is equal to the energy per unit leaving the patch.